The Impact of Wage Premiums on Educational Attainment and Social Mobility*

Niclas Frederic Poitiers†

November 13, 2018

You can find the most recent version here:
`http://nfpoitiers.eu/inequality_social_mobility`

Abstract

This paper investigates the role that wage premiums play for educational attainment and intergenerational social mobility. An important difference between countries with low and high levels of social mobility is the extent of upward mobility of children from low income families. This is mainly explained by the probability of high school dropout. I develop a model with three levels of education in which children facing a credit constraint choose which level of education to attain based on a transfer that they receive from their parents. I find in an empirical exercise that in the U. S. the opportunity cost of education is more important in explaining the high school dropout rate of men than the return on education. The model and the empirical results imply that a policy that reduces the opportunity cost of education and is paid by higher taxation on graduates, reducing the return on education, could decrease dropout rates, and also increase the number of graduates not facing a binding credit constraint. Such a policy could also be effective in increasing the college graduation rate of poor students and in decreasing levels of student debt.

Keywords: Inequality, Education, Intergenerational Mobility

JEL Classifications: E24, J31, I24

---

*I am gratefull for the comments from Xavier Raurich, Concepció Paxot, Matthias Doepke, Joseph Zeira, Vahagn Jerbashian, Aydan Dogan, and Montserrat Vilalta Bufí, and the feedback from participants at the UB Macroeconomic Seminar, the UB School of Economics Ph. D. Workshop, and the Northwestern Macro Lunch Seminar.

†Universtitat de Barcelona School of Economics. Address: John M. Keynes, 1-11, 08034 Barcelona, Spain. Email: n.f.poitiers@ub.edu

1 Introduction

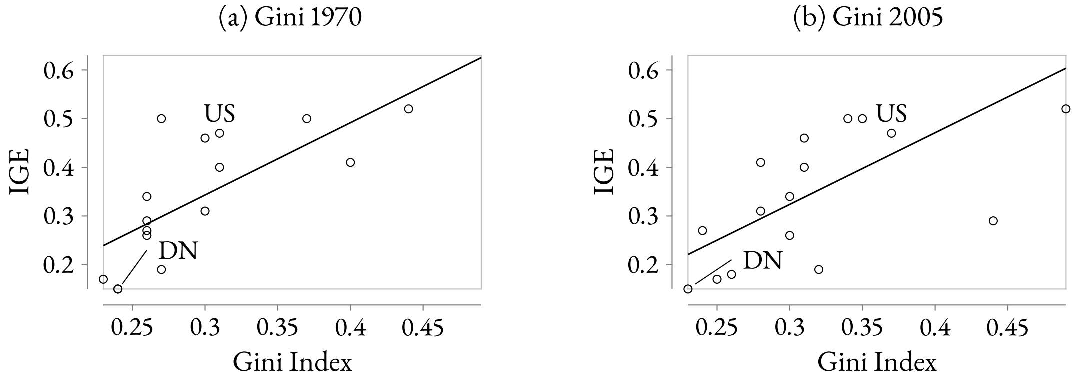
There is a strong correlation in cross country data between income inequality and intergenerational social mobility. Empirical studies suggest that this is driven by the extent in which children with parents in the highest and lowest income quintiles are downward and upward mobile over generations, respectively. One of the most important drivers of social mobility is education. I investigate in this paper to which extent income inequality, in particular wage differences, is driving differences in intergenerational social mobility. This is of particular interest as income inequality and employment polarisation are increasing in developed countries. In the literature on intergenerational mobility, inequality hinders intergenerational mobility mainly by imposing binding credit constraints on the ability of parents to acquire education for their children. The parents decide how much education to buy form their children. The ability to buy education is constrained by the budget of the parents. In this paper I explore a different approach: Not the parents but the children themselves decide on how much education to acquire They decide whether to educate or to work and earn an unskilled wage. Children from poor backgrounds can still choose to educate, but since they receive less financial support from their parents, the value of the outside option to work is much more important for them. This approach allows me to identify the effects of education finance policies on the educational outcomes of children from poorer and richer family backgrounds. The model as well as the empirical exercise suggest that opportunity cost to education play a very important role in explaining education outcomes. They are relatively more important than the returns to education. This has important implications for the effect of wage polarisation and for the way education should be payed for. The second contribution of this paper to the existing literature is that it directly models high school drop out. High school drop out is one of the main factors explaining differences in social mobility. Most models so far focus on the role of college education. Evidence in the empirical literature suggests though that high school dropout plays a more important role in explaining persistence of education levels.

The degree of income inequality in a country is negatively associated with intergenerational mobility. The more unequal a country, the more persistent is income over generations. This is called the “Great Gatsby Curve”. One measure of intergenerational social mobility is the intergenerational earnings elasticity (IGE). It is the elasticity between a child’s and their parents’ income. A higher IGE means that children’s incomes depend stronger on their parents’ income, i. e. lower intergenerational income mobility. Corak (2013a,b) estimates the IGE for a variety of countries. In Table 1, I display his IGE estimations and a variety of indices of income inequality (including some based on transition matrices of Jäntti et al., 2006). The correlation of the IGE with inequality indices are shown in Table 2.

The IGE features a high positive correlation with all of them. As a higher IGE implies less intergenerational income mobility, this implies a negative relationship between income inequality and intergenerational income mobility. The 1970 Gini-index and 2005 Gini-index have correlations of 0.69 and 0.71 with the IGE, respectively. The IGE and Gini-index relation is depicted in Figure 1. One can see the strong negative association of inequality and social mobility, the “Great Gatsby Curve”. Chetty et al. (2014a) find such a relation also within the U. S. Areas with a lower share of middle-class residents have lower levels of social mobility. Thus regions with higher levels of income inequality tend to have lower levels of intergenerational income mobility.

In order to better understand this relationship, it is helpful to look at transition matrices, which provide

Figure 1: *The Great Gatsby Curve*



Note: IGE values from Corak (2013b) and mean Gini index estimates from Standardized World Income Inequality Database (SWIID), see Table 1. The lines represent fitted values. The values for the U. S. and Denmark are highlighted, of which the transition matrices are compared in Table 2.

a more detailed picture than IGE estimates. For each income quantile, they give the probabilities for a child born into a family in this quantile to end up in each income quantile. Jäntti et al. (2006) compares the transition matrices of Scandinavian countries with the ones of the U. K. and the U. S. Jäntti et al. (2006) show that the most important dimension in which countries with high intergenerational mobility (Scandinavian countries) differ from ones with low intergenerational mobility (U. S. and U. K.) is the persistence of high and low income families (top and bottom 20%) in their quintiles. Figure 2 shows a representation of the transition matrices of Denmark and the U. S. estimated by Jäntti et al. (2006). Each line represents the probabilities for a child from one income quintile of ending up in each income quintile. One can see that in the U. S., high income families have a lower downward mobility and low income families have a lower upward mobility than in Denmark (see also Table 3).

The view that the bottom and the top of the income distribution are responsible for the relationship between income inequality and social mobility is supported by Corak et al. (2014). They observe that the main differences in absolute earnings-mobility (a son's income relative to his father's) between the U. S., Canada, and Sweden are in the extent of downward mobility (sons earning less than their fathers) from the top of the income distribution. Furthermore, Couch and Lillard (2004) observe non-linear patterns in income persistence for the U. S. and Germany.[1] They find evidence that earnings are more persistent over generations for high income families than for those with lower income.

Educational attainment determines to a large extent one's lifetime income level.[2] It is also strongly dependent on parental background. In the U. S., children from affluent families have a much higher probability of graduating from high school and a much lower probability of dropping out of high school. In Table 4, I show the educational attainment by parental income in the U. S. using the National Longitudinal Survey of Youth 1979 (NLSY-79). One can see that the probability of dropping out of high school is very low for children of affluent families (0.08), while it is very high for children from the lowest income quintile (0.36). This resembles the pattern observed in the transition matrices. The probabilities to end up in the lowest income quintile is very similar: 0.07 for children of high income families and 0.4 for children of low income families. As in the transmission of income, the difference between the top and the bottom of the income

[1] Earnings of sons are stronger related to those of their fathers when the father is richer.

[2] For a discussion of U. S. wage premiums see Lemieux (2006)

Table 1: *Measures of Inequality and Mobility*

	Gini 1970	Gini 2005	90/10	90/50	50/10	IGE	I_λ	I_{Trace}	I_{Cross}
Denmark	0.24	0.23	2.8	1.5	1.8	0.15	0.81	0.93	0.35
Finland	0.25	0.26	3.2	1.7	1.9	0.18	0.80	0.93	0.35
Norway	0.23	0.25	3.1	1.6	1.9	0.17	0.78	0.92	0.34
Sweden	0.26	0.24	3.5	1.7	2.1	0.27	0.78	0.92	0.34
United Kingdom	0.27	0.35	4.4	2.0	2.2	0.50	0.79	0.93	0.34
United States	0.31	0.37	5.8	2.2	2.7	0.47	0.66	0.87	0.30
Australia	0.26	0.30	4.4	1.9	2.3	0.26			
Canada	0.27	0.32	4.5	1.9	2.3	0.19			
Chile	0.44[1]	0.49	9.0	3.3	2.7	0.52			
France	0.40	0.28	3.5	1.9	1.9	0.41			
Germany	0.30	0.28	3.7	1.8	2.0	0.31			
Italy	0.37	0.34	4.2	1.9	2.2	0.50			
Japan	0.26	0.30	5.3	2.0	2.7	0.34			
New Zealand	0.26	0.33	4.3	1.9	2.2	0.29			
Spain	0.31	0.31	4.8	2.0	2.5	0.40			
Switzerland	0.30[2]	0.31	3.4	1.8	1.9	0.46			

NOTE: Gini index estimates are the mean estimates from the SWIID. Other measures of inequality are form the OECD, based on the year 2009, except for Australia with values for the year 2008. IGE estimates are from Corak (2006). 90/10, 90/50 and 50/10 are the ratios between the corresponding income percentiles. Measures of mobility are based on transition matrices based on the age-corrected transition matrices of father and sons from Jäntti et al. (2006). The indices based on the transition matrices are the following. The first index is based on the second largest eigenvalue λ_2 of the mobility matrix: $I_\lambda = 1 - |\lambda_2|$. In the following m is the number of rows of the matrix and p_{ij} is the transition probability from quantile i to quantile j. π_i is the long run probability of being in quantile i (i. e. $\frac{1}{m}$). The second index of mobility in transitions matrices is based on the trace of the matrix: $I_{\text{Trace}} = \frac{m - \sum_{i=1}^{m} p_{ii}}{m-1}$. The third index is based on the expected number of income brackets crossed: $I_{\text{Cross}} = \frac{1}{m-1} \sum_{i=1}^{m} \sum_{j=1}^{m} \pi_i p_{ij} |i-j|$. For all three transition matrix indices, higher values indicate lower levels of intergenerational mobility.

[1] 1968

[2] 1971

distribution is especially strong. That raises the question of how educational decisions at the top and the bottom of the income distribution are influenced by income inequality. I want to address this question by looking at wage premiums and the opportunity cost of education.

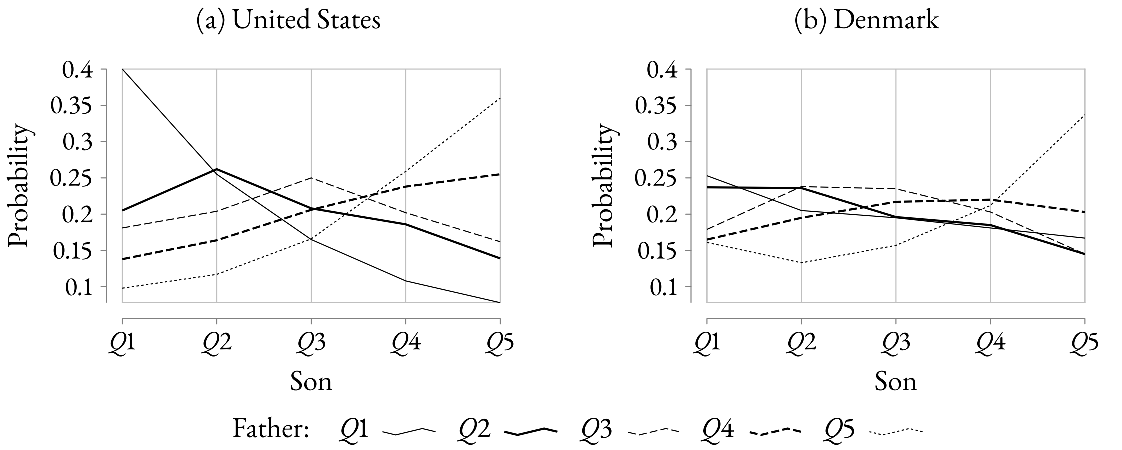
It seems to be of particular importance to understand the role that wage premiums play for intergenerational mobility, since income inequality has increased over the last decades (see Piketty, 2013) and this increase has taken the form of a polarisation of labour markets. In the last two decades there was a polarisation of the labour market, with less middle wage jobs and more low and high wage jobs. Between 1993 and 2010 the share of hours worked in middle wage occupations declined by 8 percentage points in Europe and by 6 percentage points in the U. S. (Goos et al., 2009; Acemoglu and Autor, 2011). Acemoglu and Autor (2011) found for the U. S. that during the same period, the wage growth in middle wage occupations has lagged considerably behind the wage growth in low and high wage occupations. This implies not only a polarisation of the labour market, but also of wages. Deschênes (2001), Lemieux (2006) and Acemoglu and Autor (2011) observe a convexificiation of the premiums on education since 1980, i. e. a strong increase in the return on higher education. There is no evidence of a polarisation of wages in Europe yet. There is an increase of upper tail inequality, but no decrease of lower tail inequality in the U. K. and Germany (Man-

Table 2: *Correlations of IGE and Inequality Measures*

	Gini 1970	Gini 2005	90/10	90/50	50/10	IGE
Gini 1970	1.00	0.67	0.61	0.72	0.31	0.69
Gini 2005	0.67	1.00	0.92	0.94	0.71	0.71
90/10	0.61	0.92	1.00	0.97	0.84	0.56
90/50	0.72	0.94	0.97	1.00	0.71	0.62
50/10	0.31	0.71	0.84	0.71	1.00	0.47
IGE	0.69	0.71	0.56	0.62	0.47	1.00

Note: Gini index estimates are the mean estimates from the SWIID (see Table 1). Other measures of inequality are from the OECD, based on the year 2009, except for Australia with values for the year 2008. IGE estimates are taken from Corak (2006).

Figure 2: *Transition Probabilities*



Note: Probability of a son with a father in income quintile Q_i to be in income quintile Q_j. The probabilities are from Jäntti et al. (2006), see Table 3.

ning et al., 2007; Antonczyk et al., 2010).[3] Yet, analogous to the convexification of educational returns in the U. S., Pereira and Budría (2005) and Lindley and Machin (2011) showed that the inequality in post-graduate wages in the EU has increased.

There is not only evidence of a polarisation of employment and education premiums, but also of increased polarisation of educational efforts. Putnam et al. (2012) find a growing gap between high school students from upper and middle class backgrounds with respect to participation in soft-skill building activities.[4] Ramey and Ramey (2010) observe a considerable increase in the time spent with children for middle and upper class parents since the mid-1990s, and Kornrich and Furstenberg (2013) show that the investment into children's education is increasingly unequal. Bailey and Dynarski (2011) find increasing dependence of college attendance on income for the period of 1961 to 1982, driven by an increase in the college attendance of daughters of high income families, but this has stabilised after 1982 (Chetty et al., 2014b). Lindley and Machin (2012) find for the U. K. that with increasing length of education, the importance of the family background is increasing.

Although the pattern of increased income inequality and convexification of the return on education

[3] Here, upper tail inequality relates to the ration of the 90th to the 50th income percentile and lower tail inequality to the ration of the 50th to the 10th income percentile.

[4] The gap in participation in extracurricular activities, i. e. sports and academic clubs, is increasing. These participations are a strong predictors of future success (Putnam et al., 2012).

Table 3: *Transition Matrices*

		Denmark					U. K.				
		Son					Son				
		Q_1	Q_2	Q_3	Q_4	Q_5		Q_1	Q_2	Q_3	Q_4	Q_5
Father	Q_1	0.253	0.205	0.195	0.181	0.167	Q_1	0.303	0.235	0.165	0.174	0.122
	Q_2	0.237	0.236	0.196	0.185	0.145	Q_2	0.241	0.227	0.182	0.193	0.157
	Q_3	0.179	0.238	0.235	0.203	0.145	Q_3	0.188	0.195	0.227	0.206	0.184
	Q_4	0.165	0.195	0.217	0.220	0.203	Q_4	0.161	0.175	0.229	0.195	0.240
	Q_5	0.161	0.133	0.157	0.212	0.337	Q_5	0.107	0.168	0.197	0.231	0.297

		U. S. NLSY-79					U. S. Chetty					
		Son						Child				
		Q_1	Q_2	Q_3	Q_4	Q_5			Q_1	Q_2	Q_3	Q_4	Q_5
Father	Q_1	0.400	0.254	0.165	0.108	0.074	Parents	Q_1	0.337	0.242	0.178	0.134	0.109
	Q_2	0.205	0.262	0.208	0.186	0.139		Q_2	0.280	0.242	0.198	0.160	0.119
	Q_3	0.181	0.204	0.250	0.202	0.162		Q_3	0.184	0.217	0.221	0.209	0.170
	Q_4	0.138	0.164	0.206	0.238	0.255		Q_4	0.123	0.176	0.220	0.244	0.236
	Q_5	0.098	0.117	0.166	0.259	0.360		Q_5	0.075	0.123	0.183	0.254	0.365

NOTE: Transition matrices from Jäntti et al. (2006), income quintile transition matrices for sons and fathers, corrected for age. U. S. data from Chetty et al. (2014a) generally linking parents to children (both sons and daughters).

point in the direction of less intergenerational social mobility, there is no agreement in the literature yet on whether intergenerational social mobility has actually decreased as a consequence. To the best of my knowledge, there are only attempts to estimate trends in social mobility for the U. S. Point in time measurements indicate a strong increase of the IGE. Estimates of the IGE for the 1960s to 80s are around 0.2 (Becker and Tomes, 1986), Solon (1999) estimates it for the 1990s at around 0.4 and current estimates are around 0.6 as in Mazumder (2005).[5] Aaronson and Mazumder (2008) estimate the U. S. trend of the IGE based on Integrated Public Use Microdata Series (IPUMS). The child's year and state of birth are used to construct predicted parents' incomes. Their IGE estimations track the upward trend in income inequality between 1970 to 2000 very closely. Hertz (2007) and Lee and Solon (2008) estimate IGE trends based on the Panel Study of Income Dynamics (PSID) over the same time period. The estimations show no clear trend in the IGE.[6] Chetty et al. (2014b) estimate social mobility indices based on de-identified tax records and college attendance rates. They find no trend in social mobility measurements based on income rank, but as income inequality increases, the consequences of rank mobility has increased.[7]

In this paper, I develop an overlapping generations model (OLG model) based on Galor and Zeira (1993), with three levels of education where children choose their education level based on a transfer that they receive from their parents. The three levels of education reflect the different intergenerational mobility patterns of poor, middle class, and rich families observed in the empirical literature. Letting the children instead of the parents choose the education level allows to identify opportunity costs of education that are relevant for children form poor families but not for children from rich families. In Galor and Zeira (1993) and the

[5]A simple comparison of estimates with different samples and life-time income definitions suffers form comparability problems. For a discussion, see Hertz (2007).

[6]Lee and Solon (2008) observe a small increase for daughters. Hertz (2007) does estimations with four different specifications. In one he finds a positive trend in IGE, however, the other specifications show no trend at all.

[7]The probability of a child born into a family at the lowest quintile of the income distribution to reach the highest quintile is 8.4% for the 1971 cohort and 9% for the 1986 cohort.

related literature, parents choose the education level based on the costs of education for them. Parents facing a binding credit constraint cannot afford higher education for their children. In this paper children also face a credit constraint, but even if this credit constraint is binding they can choose a low level of consumption in the first part of their lives in order to acquire higher education and have a higher income in the future. Poor children cannot smooth consumption over both periods. The alternative of working instead of studying is therefore of higher importance for them then for rich children.

I find in the model that an increase in the opportunity cost decrease educational attainment by the poor, whereas an increase in the return on education increases educational attainment overall. Furthermore, when keeping the transfer from the parents unchanged, changes in the wages of graduates have an impact on the number of graduates not facing a binding credit constraint, whereas changes in the wages of dropouts of education just affect the number of graduates facing a binding credit constraint. In an empirical assessment of the relative importance of the opportunity costs of education using U. S. data, I find that changes in the opportunity cost have a much stronger influence on the high school dropout rate of men than the return on education. Therefore I propose a policy where the costs of education are paid by graduates in later stages of their life through taxes, which reduces the return on education but decreases the opportunity cost of education. This policy has the advantage, that it does not imply transfers between educational groups, decreases income inequality due to age differences, and increases the number of graduates as well as the number of graduates not facing a binding credit constraint. Furthermore this paper suggests that policies affecting the income distribution should not only be assessed in their effect on the overall level of income inequality, but also in how these policies affect the incentives for educational attainment, in particular the opportunity cost of education.

Current models of social mobility do mainly explain the impact of income inequality on social mobility through credit constraint agents (Galor and Zeira, 1993). Only rich agents with income or wealth above a certain threshold acquire education. Higher income inequality implies that more persons are below the threshold for acquiring education or, as in the case of Moav and Galor (2004), can acquire an optimal level of education.[8] Alonso-Carrera et al. (2012) further develop the model of Galor and Zeira (1993) in order to allow for fiscal policies. In their model, using labour taxes instead of inheritance taxes increase human capital accumulation, while the impact of such a policy on income inequality would depend on the initial distribution of human capital. According to the analysis of Jäntti et al. (2006), it is not the middle class at the threshold of affording education that is responsible for lower mobility, but the upper and lower class, which in these models would always be above or below the income threshold.[9] Piketty (1995) and Checchi et al. (1999) model private and public investment into human capital as the results of beliefs about ability and effort. Differences in social mobility are the outcomes of differences in experienced mobility of dynasties. Both the theories of Piketty (1995) and Checchi et al. (1999) do imply that the differences in social mobility are a result of long term differences (over several generations) in countries' economic structures.

In Galor and Tsiddon (1997) and Hassler and Rodríguez Mora (2000), income inequality and social mobility depend on the rate of technological progress. In their models, technological progress increases

[8]Galor and Moav (2006) argue that the increase in public education after the industrialisation was not the result of class struggles, but a result of an interest of the capitalist class in educated workers. In this model, the binding credit constraint is not overcome by a sufficient decrease of inequality, but by a political interest in taxation of the upper class.

[9]And as Chetty et al. (2014a) point out, the middle class is actually the most mobile class, its lacking is impeding mobility strongly.

inequality, as it increases the return on skill, but it also decreases the role of inherited human capital and thus increases social mobility. The evidence presented here goes the other way round: more income inequality is linked to less intergenerational social mobility, not more. Alonso-carrera et al. (2016) use a framework similar to Galor and Tsiddon (1997) and Hassler and Rodríguez Mora (2000), but come to the opposite conclusion. They study the interaction between the education decision and the choice of occupations with different effort levels. An increase in the return on effort for high skilled decreases the frequency of high wages of low skilled, potentially increasing income inequality while reducing intergenerational social mobility. In Hassler et al. (2007) income inequality affects social mobility through two channels: through its effect on incentives for education and through its effect on parents ability to pay for their children's education. They emphasise the role of public education for mitigating the latter effect.

There are some studies on the reasons for high school dropout, but they are not considering social mobility. Eckstein and Wolpin (1999) study the causes of dropouts from high school using the NLSY-79. Students that drop out of high school have lower expectations about the reward of graduating.[10] McNeal (1997) studies the effects of employment during high school on the probability of dropping out of high school. The job type and the intensity have strong effects on the probability of dropping out of high school. The study most similar to my approach is Restuccia and Urrutia (2004). They developed an OLG model distinguishing between college and early (i. e. pre-college) education. They argue that about half of intergenerational income persistence is due to parental investments into early education, but college education accounts for most of the disparity.[11]

My approach contributes to this literature by studying the influence of wage differentials for educational choices. To this end, I introduce the decision of high school drop out into a Galor and Zeira (1993) framework. I developed an OLG model that captures the education decision of agents with respect to both high school and college. The agents make their choice regarding their individual opportunity cost of education and the return on education. In contrast to Restuccia and Urrutia (2004), I focus on the role of inequality by directly modelling the educational choice in pre-college education. As the evidence presented above indicates, this pre-college educational choice is crucial for understanding differences in social mobility. The second contribution of my model is that it distinguishes between the cost of education for parents and the incentive for children to graduate, i. e. the cost of education for the children in terms of utility. Children from low income families have a lower level of consumption as students, and dropping out of high school, i. e. giving up future income for current income, is a much more attractive option. On the other side of the income distribution, children from rich families already enjoy a high level of consumption during education and thus there is no incentive to drop out of high school.

The rest of the paper is structured in the following way: Section 2 introduces the model, Section 3 explains the educational choice of agents in the model, Section 4 explores the baseline comparative statics in the model, in Section 5 I make an empirical assessment of the relative importance of wage premiums, and in Section 6 I discusses the effectiveness of policies.

[10] Eckstein and Wolpin (1999) argue that a prohibition of working for high school students would have only limited impact, as the traits the children have when they come to school play an important role.

[11] Early education accounts for the largest part of the persistence, as younger parents are more strongly constrained in their budget for educational expenses.

Table 4: *High School Dropouts*

	Total	Q_1	Q_2	Q_3	Q_4	Q_5	Q_1	Q_2	Q_3	Q_4	Q_5
HS dropout[1]	1511	474	425	295	209	108	0.36	0.32	0.23	0.16	0.08
HS[2]	1906	408	397	412	392	297	0.31	0.30	0.31	0.29	0.22
HS+[3]	3200	436	501	603	731	929	0.33	0.38	0.46	0.55	0.70
Total	6617	1318	1323	1310	1332	1334	1.00	1.00	1.00	1.00	1.00

NOTE: Education level by parents' income quintiles. The data are from the NLSY-79 containing 14-22 year olds in 1979. Income quintiles correspond to the mean reported values before the age of 18.
[1] Highest attended grade <12.
[2] Highest grade attended 12.
[3] At least some college education.

2 The Model

I propose an OLG model based on Galor and Zeira (1993) of intergenerational social mobility in which agents decide which level of education to acquire. The model consists of three different levels of education in order to model the different mobility patterns observed for low, middle an high wage groups. This allows me to study the effect of wage polarisation on educational attainment. The education decision is defined by two forces: the opportunity cost of education and the return on education. The former are only relevant for children from poor backgrounds, whereas the latter also matter for rich children. Each individual lives for three periods. Each individual has one parent and one child, thus there is no population growth. The timing in the three periods of life is as follows: In the first period, the agent is born, he goes to school and can decide to drop out in order to earn the wage of unskilled workers for this period and the rest of his life. He can decide to finish school and work for the rest of his life for the high school graduate wage, or he can go to college, and does not earn anything in this period, but earns a college wage for the rest of his life. In the second period, the agent works and gets one child. He provides a transfer to his child. In the third period, the agent retires and consumes from his savings.

In the following, the index N denotes drop out of high school, H high school graduation and C college graduation. Depending on his education level, the agent has the following incomes in the first period of his life:

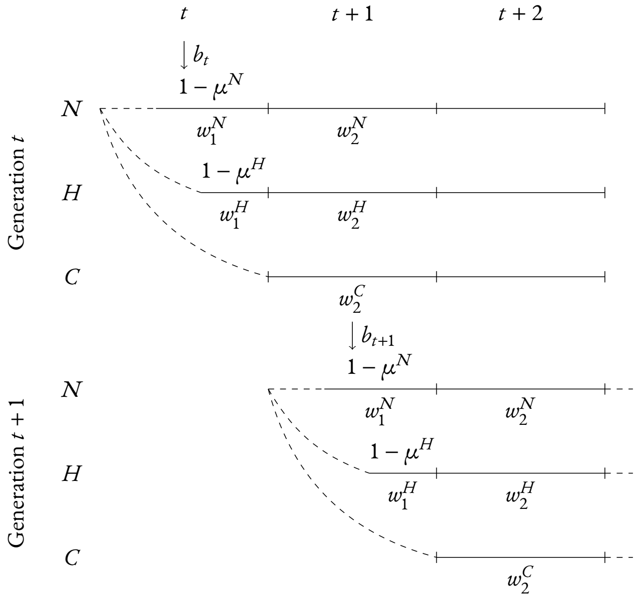
$$y_{1,t} = \begin{cases} (1-\mu_N)w_{N,1} & \text{Dropout} \\ (1-\mu_H)w_{H,1} & \text{High School} \\ 0 & \text{College} \end{cases}$$

where $w_{N,1} < w_{H,1}$ and $0 < \mu_N < \mu_H < 1$, and in the second period,

$$y_{2,t+1} = \begin{cases} w_{N,2} & \text{Dropout} \\ w_{H,2} & \text{High School} \\ w_{C,2} & \text{College} \end{cases}$$

where $w_{N,2} < w_{H,2} < w_{C,2}$. Here μ_N denotes the time devoted to education in the first period of one's life if one drops out of high school, and μ_H the time devoted to education if one graduates from high school.

Figure 3: *Generation Overlap*



NOTE: Generation overlap and incomes by education level in the model.

$w_{X,i}$ is the wage level of an agent with an education level X in period i of his life. I assume $w_{N,1} < w_{H,1}$ and $w_{N,2} < w_{H,2} < w_{C,2}$ in order to have productivity increase with education. $\mu_N > \mu_H$ implies that by dropping out of high school, a young person has more time to work during their youth. I assume

$$(1 - \mu_N)w_{N,1} > (1 - \mu_H)w_{H,1} \tag{1}$$

in order to incentivise drop out of high school.

The utility of the agent is of the following form:

$$U = \log(c_{1,t}) + \beta \log(c_{2,t+1}) + \beta^2 \log(c_{3,t+2}) + \gamma\beta \log(b_{t+1}), \tag{2}$$

where $\beta \in (0, 1)$ is the subjective discount rate and $c_{1,t}$, $c_{2,t+1}$, and $c_{3,t+2}$ are, respectively, the consumption levels in the first, second, and third period of life. The parents are altruistic to wards their children in a joy of giving way and γ captures the degree of altruism to wards the child. b_{t+1} is a transfer from the parent to the child in the first period of the child's life.

The agent can save in order to transfer income to later periods. $s_{i,t}$ denotes the savings that carry income from period i to period $i + 1$ and $r \in (0, 1)$ is the corresponding interest rate on saving, with $R = 1 + r$. In the first period, the agent receives the transfer b_t from his parents and earns net income $(1 - \tau)y_{1,t}$, where τ is the income tax rate. If he acquires an education level X, he has to pay the cost e^X of his education, which is subsidised by the state in the height of h^X. In order to simplify notation, I define $g_1^X = e^X - h^X$. He uses the rest of his income for consumption $c_{1,t}$ and savings $s_{1,t}$. In the second period, he gets net-income $(1 - \tau)y_{2,t+1} - g_2^X$, where g_2^X is an additional tax to induce progressive taxation, and first period savings $s_{1,t}$. g_1^X and g_2^X depend on the education level X in order to introduce progressive taxation and education specific costs and subsidies. The agent uses this for consumption $c_{2,t+1}$, savings $s_{2,t+1}$ and the transfer b_{t+1} to his child. The retired agent consumes all of his savings in the third period. Hence, one gets the following budget constraints:

$$c_{1,t} + s_{1,t} = b_t + (1-\tau)y_{1,t} - g_1^X, \quad (3)$$

$$c_{2,t+1} + b_{t+1} + s_{2,t+1} = (1-\tau)y_{2,t+1} - g_2^X + Rs_{1,t}, \quad (4)$$

and

$$c_{3,t+2} = Rs_{2,t+1}. \quad (5)$$

The general idea is that agents face a trade off between income in the first period of their lives and higher income in the second period of their lives due to higher education. Agents face a credit constraint $s_{1,t} \geq 0$, thus if their parents do not provide them with a large enough transfer b_t, they cannot optimise their utility over both periods. Thus, agents that receive a small transfer from their parents face a trade of between higher first period consumption and low education, and lower first period consumption and higher education. In order to induce this trade off I assume that life-time income is increasing with education:

$$(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - Rg_1^N - g_2^N < (1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - Rg_1^H - g_2^H, \quad (6)$$

and

$$(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - Rg_1^H - g_2^H < (1-\tau)y_{2,t+1}^C - Rg_1^C - g_2^C. \quad (7)$$

The problem of the agent is to maximise utility (2) subject to (3) - (5). From the first order condition, I obtain the optimal consumption, transfer, and savings for unconstrained agents with $s_{1,t} \geq 0$ and education X:

$$c_{1,t}^{X*} = \frac{1}{R(1+\beta+\beta^2+\beta\gamma)}\left[(1-\tau)(Ry_{1,t}+y_{2,t+1}) - g_2^X + R(b_t - g_1^X)\right], \quad (8)$$

$$c_{2,t+1}^{X*} = \frac{\beta}{1+\beta+\beta^2+\beta\gamma}\left[(1-\tau)(Ry_{1,t}+y_{2,t+1}) - g_2^X + R(b_t - g_1^X)\right], \quad (9)$$

$$c_{3,t+2}^{X*} = \frac{R\beta^2}{1+\beta+\beta^2+\beta\gamma}\left[(1-\tau)(Ry_{1,t}+y_{2,t+1}) - g_2^X + R(b_t - g_1^X)\right], \quad (10)$$

$$b_{t+1}^{X*} = \frac{\beta\gamma}{1+\beta+\beta^2+\beta\gamma}\left[(1-\tau)(Ry_{1,t}+y_{2,t+1}) - g_2^X + R(b_t - g_1^X)\right], \quad (11)$$

and

$$s_{1,t}^{X*} = (1-\tau)y_{1,t} - g_1^X + b_t - c_{1,t}^{X*}. \quad (12)$$

As the model features homothetic preferences, in absence of the credit constraint the maximisation of utility is equivalent to the maximisation of life-time income, which can be seen in (8) – (11). Therefore, if he is not credit constrained, the agent always prefers to have a college education. This follows from (6) – (7). He only drops out of education because he faces a credit constraint $s_{1,t} \geq 0$, which depends on b_t, as follows from (12). Because the income depends on the education decision, I obtain three different thresholds for the transfer $\{\hat{b}_t^N, \hat{b}_t^H, \hat{b}_t^C\}$, below which the agent is facing a binding credit constraint:

$$\begin{aligned}
\hat{b}_t^N &= \frac{1}{R(\beta+\beta^2+\beta\gamma)}\left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - Rg_1^N - g_2^N\right] \\
&\quad - \frac{(1+\beta+\beta^2+\beta\gamma)}{\beta+\beta^2+\beta\gamma}\left[(1-\tau)y_{1,t}^N - g_1^N\right], \\
\hat{b}_t^H &= \frac{1}{R(\beta+\beta^2+\beta\gamma)}\left[(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - Rg_1^H - g_2^H\right] \\
&\quad - \frac{(1+\beta+\beta^2+\beta\gamma)}{\beta+\beta^2+\beta\gamma}\left[(1-\tau)y_{1,t}^H - g_1^H\right],
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
\hat{b}_t^C &= \frac{1}{R(\beta+\beta^2+\beta\gamma)}\left[(1-\tau)y_{2,t+1}^C - Rg_1 C - g_2^C\right] \\
&\quad - \frac{(1+\beta+\beta^2+\beta\gamma)}{\beta+\beta^2+\beta\gamma} g_1^C.
\end{aligned} \tag{14}$$

It follows from (1), (6) & (7) that $\hat{b}_t^N < \hat{b}_t^H < \hat{b}_t^C$. Thus the higher the education level, the higher the transfer needed in order to not face a binding credit constraint.

The consumption levels of a credit constrained agent are:

$$\begin{aligned}
\bar{c}_{1,t}^X &= (1-\tau)y_{1,t} - g_1^X + b_t, \\
\bar{c}_{2,t+1}^X &= \frac{\beta}{\beta+\beta^2+\beta\gamma}\left[(1-\tau)y_{2,t+1} + g_2^X\right], \\
\bar{c}_{3,t+2}^X &= \frac{R\beta^2}{\beta+\beta^2+\beta\gamma}\left[(1-\tau)y_{2,t+1} - g_2^X\right],
\end{aligned}$$

and

$$\bar{b}_{t+1}^X = \frac{\beta\gamma}{\beta+\beta^2+\beta\gamma}\left[(1-\tau)y_{2,t+1} - g_2^X\right].$$

Thus an agents who receives a transfer below $\hat{b}_t^N$ will always face a binding credit constraint, irrespective of education, an agent who receives a transfer between $\hat{b}_t^N$ and $\hat{b}_t^H$ can either drop out of high school and be not bound by the credit constraint or graduate from high school and face a binding credit constraint, an agent who receives a transfer between $\hat{b}_t^H$ and $\hat{b}_t^C$ can either work after high school and not be bounded by the credit constraint or graduate from college and face a binding credit constraint, and an agent receiving a transfer above $\hat{b}_t^C$ can graduate from college without facing a binding credit constraint.

3 Education Decision

Agents are identical in this model except for their parental background, i. e. the transfer b_t which they receive from their parents. Thus, their decision on which level of education to attain will only depend on b_t. If they attain higher education, they will have a higher second period income but a lower first period income. This matters only if the agents are credit constrained. I will first consider the decision between high school graduation and drop out of high school and later the decision between high school graduation and college education.

In order to determine whether it is optimal for a young agent to graduate or to drop out of high school, I consider the value of these two option in terms of utility. I define $V^N(b_t)$ as the utility when dropping out and $V^H(b_t)$ as the utility when graduating from high school. When the agent receives a $b_t \in [\hat{b}^N, \hat{b}^H)$ he would face a binding credit constraint when graduating from high school, but not when dropping out. The value of dropping out of high school in terms of utility is

$$V^N(b_t) = \log c_{1,t}^{N*} + \beta \log c_{2,t+1}^{N*} + \beta^2 \log c_{3,t+2}^{N*} + \beta\gamma \log b_{t+1}^{N*},$$

with $c_{1,t}^{N*}$, $c_{2,t+1}^{N*}$, $c_{3,t+2}^{N*}$, and b_{t+1}^{N*} being the consumption and transfer obtained when he dropped out from school. This can be rewritten as

$$\begin{aligned} V^N(b_t) &= (1+\beta+\beta^2+\beta\gamma)\log\left(\frac{1}{1+\beta+\beta^2+\beta\gamma}\right) \\ &+ \left[\log\left(\frac{1}{1+r}\right) + \beta\log(\beta) + \beta^2\log(R\beta^2) + \beta\gamma\log(\beta\gamma)\right] \\ &+ (1+\beta+\beta^2+\beta\gamma)\log\left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t - g_1^N)\right]. \end{aligned}$$

The value of graduating from high school in terms of utility is equal to

$$\bar{V}^H(b_t) = \log \bar{c}_{1,t}^H + \beta \log \bar{c}_{2,t+1}^H + \beta^2 \log \bar{c}_{3,t+2}^H + \beta\gamma \log \bar{b}_{t+1}^H,$$

with $\bar{c}_{1,t}^H$, $\bar{c}_{2,t+1}^H$, $\bar{c}_{3,t+2}^H$ and $\bar{b}_{t+1}^H$ being the consumption and transfer of a credit constraint agent with high school income. This can be rewritten to

$$\begin{aligned} \bar{V}^H(b_t) &= (\beta+\beta^2+\beta\gamma)\log\left(\frac{1}{\beta+\beta^2+\beta\gamma}\right) \\ &+ \left[\log(y_{1,t}^H - g_1^H + b_t) + \beta\log\beta + \beta^2\log(R\beta^2) + \beta\gamma\log(\beta\gamma)\right] \\ &+ (\beta+\beta^2+\beta\gamma)\log\left[(1-\tau)y_{2,t+1}^H - g_1^H\right]. \end{aligned}$$

I am looking for the value of the transfer $b_t^H \in [\hat{b}_t^N, \hat{b}_t^H)$ above which the agent finishes high school and below which he drops out of high school. I obtain this cutoff value by looking at the b_t for which the agent is indifferent between graduating from high school and dropping out, i. e. for which the values of the two value functions are equal. I get for the transfer b_t^H

$$V^N(b_t^H) = \bar{V}^H(b_t^H).$$

This can be reformulated as

$$\log\left\{\frac{(1-\tau)(Ry_{1,t}^{N}+y_{2,t+1}^{N})-g_{2}^{N}+R(b_{t}-g_{1}^{N})}{R(1+\beta+\beta^{2}+\beta\gamma)\left[(1-\tau)y_{1,t}^{H}-g_{1}^{H}+b_{t}\right]}\right\}=$$
$$(\beta+\beta^{2}+\beta\gamma)\log\left[\frac{1+\beta+\beta^{2}+\beta\gamma}{\beta+\beta^{2}+\beta\gamma}\frac{(1-\tau)y_{2,t+1}^{H}-g_{2}^{H}}{(1-\tau)(Ry_{1,t}^{N}+y_{2,t+1}^{N})-g_{2}^{N}+R(b_{t}-g_{1}^{N})}\right]. \tag{15}$$

I define $b_t^H \in [\hat{b}_t^N, \hat{b}_t^H)$ as the transfer that fulfils the equation above. Agents receiving a transfer above b_t^H graduate from high school, agents receiving a transfer below b_t^H drop out of high school.

I next proceed in a similar way to obtain the threshold b_t^C above which agents graduate from college. In particular, I compare the value of high school education in terms of utility for an agent not facing a binding credit constraint

$$\begin{aligned}V^{H}(b_{t})=&(1+\beta+\beta^{2}+\beta\gamma)\log\left(\frac{1}{1+\beta+\beta^{2}+\beta\gamma}\right)\\&+\left[\log\left(\frac{1}{1+r}\right)+\beta\log(\beta)+\beta^{2}\log(R\beta^{2})+\beta\gamma\log(\beta\gamma)\right]\\&+(1+\beta+\beta^{2}+\beta\gamma)\log\left[(1-\tau)(Ry_{1,t}^{H}+y_{2,t+1}^{H})-g_{2}^{H}+R(b_{t}-g_{1}^{H})\right],\end{aligned}$$

with the value of college education for an agent facing a binding credit constraint

$$\begin{aligned}\bar{V}^{C}(b_{t})=&(\beta+\beta^{2}+\beta\gamma)\log\left(\frac{1}{\beta+\beta^{2}+\beta\gamma}\right)\\&+\left[\log(b_{t}-g_{1}^{C})+\beta\log\beta+\beta^{2}\log(R\beta^{2})+\beta\gamma\log(\beta\gamma)\right]\\&+(\beta+\beta^{2}+\beta\gamma)\log\left[(1-\tau)y_{2,t+1}^{C}-g_{2}^{C}\right].\end{aligned}$$

Receiving $b_t^C \in [\hat{b}_t^H, \hat{b}_t^C)$ makes an agent indifferent between high school and college education if it fulfils the following condition:

$$V^{H}(b_t^C)=\bar{V}^{C}(b_t^C),$$

which can be reformulated as

$$\log\left[\frac{(1-\tau)(Ry_{1,t}^{H}+y_{2,t+1}^{H})-g_{2}^{H}+R(b_{t}^{C}-g_{1}^{H})}{R(1+\beta+\beta^{2}+\beta\gamma)(b_{t}^{C}-g_{1}^{C})}\right]=$$
$$(\beta+\beta^{2}+\beta\gamma)\log\left[\frac{1+\beta+\beta^{2}+\beta\gamma}{\beta+\beta^{2}+\beta\gamma}\frac{(1-\tau)y_{2,t+1}^{C}-g_{2}^{C}}{(1-\tau)(Ry_{1,t}^{H}+y_{2,t+1}^{H})-g_{1}^{H}+R(b_{t}^{C}-g_{1}^{H})}\right]. \tag{16}$$

An agent that receives a b_t larger than b_t^C will go to college, an agent which receives a b_t smaller than b_t^C will not go college and start working after graduating from high school.

Thus depending on the b_t, the agent can be in six different situations: (i) if $b_t < \hat{b}_t^N$ he drops out of high school but is still facing a binding credit constraint, (ii) if $b_t \in [\hat{b}_t^N, b_t^H)$ he drops out of high school but is not facing a binding credit constraint, (iii) if $b_t \in [b_t^H, \hat{b}_t^H)$ he graduates from high school but is facing a binding credit constraint in doing so, (iv) if $b_t \in [\hat{b}_t^H, b_t^C)$ he graduates from high school and is not facing a binding credit constraint in doing so, (v) if $b_t \in [b_t^C, \hat{b}_t^C)$ he goes to college and faces a binding credit constraint in doing so, and (vi) if $b_t \geq \hat{b}_t^C$ he graduates from college without facing a binding credit constraint.

4 Wage Polarisation and Social Mobility

In order to understand how trends in wages and policies affect intergenerational mobility, I use (15) and (16) to perform a comparative statics exercise of the transfers needed to graduate from high school and college, b^H and b^C respectively. If these minimum transfers increase, ceteris paribus, less student will receive a transfer above them and there will be less upward social mobility. If these minimum transfers decrease, more students will receive a transfer above these minimum transfers and there will be more upward social mobility. One can use that to make predictions on how changes in wage differentials affect the graduation rates of high school and college, which I will use in Section 6 to propose policies to increase graduation rates.

The choice of whether to acquire higher education or not is defined by the opportunity cost of education and the return on education. Because $b^H < b^C$, and anyone who wants to go to college needs a high school degree, I only compare the incomes of the "neighbouring" education levels, i. e. the opportunity costs and return on education of high school graduates relative to drop outs, and the opportunity cost and return on education of high school graduates and college graduates.

I define the opportunity cost of education as how much lower first period income is when spending more time on eduction. In the framework of this model, I define therefor the opportunity cost of high school education as how much higher the first period income is when dropping out then when graduating from high school. When an agent drops out of high school, he earns $y_{1,t}^N$, so the opportunity cost is this income relative to the income he would earn if he would graduate from high school, i. e. $y_{1,t}^N / y_{1,t}^H$. Since college graduates cannot work in the first period of their lives in this model, the opportunity cost of college education is the income the student could earn in the first period of his life when not going to college, i. e. the income as a young high school graduate $y_{1,t}^H$. I define the returns on education as how much higher second period income is when graduating. In the framework of this model, the return on high school education is how much relatively higher second period high school graduate income is than drop out income. If the agent drops out of high school in the first period of his life, he earns $y_{2,t+1}^N$ in the second period of his life. If he graduates from high school, he earns $y_{2,t+1}^H$. Thus the return on high school education is $y_{2,t+1}^H / y_{2,t+1}^N$. The return on college education is in this model how much higher second period income of college graduates is relative to the income of high school graduates. If the agent only graduates from high school and does not go to college in the first period of his life, he earns $y_{2,t+1}^H$ in the second period of his life. If he graduates from college instead, he earns $y_{2,t+1}^C$ in the second period of his life. Thus the return on college education is $y_{2,t+1}^C / y_{2,t+1}^H$.

In order to analyse the role of these wage premiums, I establish the following relationship between $c_{1,t}^{N*}$, $\bar{c}_{1,t}^H$, $c_{1,t}^{H*}$ and $\bar{c}_{1,t}^C$:

Lemma 1. *For an agent that receives b_t^H it is true that $c_{1,t}^{N*} > \bar{c}_{1,t}^H$ and for an agent that receives b_t^C it is true that $c_{1,t}^{H*} > \bar{c}_{1,t}^C$.*

Proof. Assume that for $b_t = b_t^H$ that $c_{1,t}^{N*} \leq \bar{c}_{1,t}^H$. Since

$$\bar{c}_{2,t+1}^H > c_{2,t+1}^{H*} > c_{2,t+1}^{C*},$$
$$\bar{c}_{3,t+2}^H > c_{3,t+2}^{H*} > c_{3,t+2}^{C*},$$

and

$$\bar{b}_{t+1}^H > b_{t+1}^{H*} > b_{t+1}^{C*},$$

this implies that $V^N(b_t) < \bar{V}^H(b_t)$. This is in contradiction to the definition of b_t^H, thus $c_{1,t}^{N*}$ has to be larger than $\bar{c}_{1,t}^H$. Using the same logic, one can derive that $c_{1,t}^{H*} > \bar{c}_{1,t}^C$. □

First, I use the implicit function theorem to get the effect of a change in the opportunity cost of high school education on the value of b_t^H. In order to simplify notation, I use in the following

$$\Gamma = \beta + \beta^2 + \beta\gamma.$$

Proposition 1. *A decrease in the opportunity cost of high school education $y_{1,t}^N / y_{1,t}^H$ decreases the transfer b_t^H needed in order to graduate from high school.*

Proof. The partial derivative of b_t^H with respect to $y_{1,t}^N / y_{1,t}^H$ is

$$\frac{\partial b_t^H}{\partial \frac{y_{1,t}^N}{y_{1,t}^H}} = -\frac{(1+\Gamma)(1-\tau)y_{1,t}^H\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right]}{(1+\Gamma)R\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right] - \left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]}.$$

It follows from Lemma 1 that this is positive. □

Thus a decrease in the opportunity cost of high school education decreases the transfer needed for graduating from high school and ceteris paribus more students will graduate. If $y_{1,t}^N / y_{1,t}^H$ increases, the relative value of the outside option to graduating from high school increases, which makes it less attractive to graduate from college for agents facing a binding credit constraint. Thus the transfer that these agents need to get in order to be indifferent between graduating from high school and dropping out has to be higher. The effect of the opportunity costs on high school education on b_t^H is the larger, the higher first period income for dropouts $y_{1,t}^N$, the higher life-time income for dropouts, and the lower the difference between the consumption level of young high school graduates and young dropouts $\bar{c}_{1,t}^H - c_{1,t}^{N*}$.

The transfer needed to graduate from high school is affected by the wage premium on high school graduation in the following way:

Proposition 2. *An increase in the return on high school education $y_{2,t+1}^H / y_{2,t+1}^N$ decreases the transfer needed in order to graduate from high school b_t^H.*

Proof. The partial derivative of b_t^H with respect to $y_{2,t+1}^H/y_{2,t+1}^N$ is

$$\frac{\partial b_t^H}{\partial \frac{y_{2,t+1}^H}{y_{2,t+1}^N}} = \frac{\Gamma(y_{2,t+1}^N - g_2^N)\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right]\left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]}{(y_{2,t+1}^H - g_2^H)\left\{(1+\Gamma)R\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right] - \left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]\right\}}$$

which following Lemma 1 is negative. □

Thus an increase in the return on high school education will lead to a decrease in the transfer needed in order to graduate from high school. If the returns on high school education increase, it becomes more attractive to graduate relative to the option of dropping out of high school. The transfer needed to be indifferent between dropping out and graduating from high school will be lower and, ceteris paribus, more students will receive a transfer above this threshold, leading to an increase in the number of students graduating from high school. The effect of the return on education on the transfer needed to graduate from high school is increasing in $\bar{c}_{1,t}^H$ and $c_{1,t}^{N*}$, and decreasing in $y_{2,t+1}^H$ and the difference $\bar{c}_{1,t}^H - c_{1,t}^{N*}$. An increase in the return on graduating from high school decreases the transfer needed in order to graduate from high school, and thus ceteris paribus increases the number of high school graduates.

The effects of changes in the wage premiums for college education can be analysed in the same way. First I look at the opportunity cost of education. As college graduates cannot work in the first period of their lives, the opportunity cost of college education is equal to $y_{1,t}^H$.

Proposition 3. *A decrease in the opportunity costs of college education $y_{1,t}^H$ decreases the transfer b_t^C needed in order to graduate from college.*

Proof. The partial derivative of b_t^C with respect to $y_{1,t}^H$ is equal to

$$\frac{\partial b_t^C}{\partial y_{1,t}^H} = -\frac{(1-\tau)Rb_t^C}{(1+\Gamma)R(b_t^C - g_1^C) - \left[(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - g_1^C + R(b_t^C - g_2^C)\right]},$$

which following Lemma 1 is positive. □

A decrease in the wage of young high school graduates decreases the value of the alternative to graduating from college. Thus going to college becomes relatively more attractive and the transfer needed to be indifferent between college and only high school education decreases. This leads to an increase in the number of agents receiving a transfer above this threshold. Equivalently to the case of high school education, the denominator is equivalent to the difference $\bar{c}_{1,t}^C - c_{1,t}^{H*}$. An increase in this difference decreases the effect of $y_{1,t}^H$, whereas an increase in b_t^C increases the effect of $y_{1,t}^H$ on itself. It has no effect on the number of college graduates that are not facing a binding credit constraint, thus an increase in $y_{1,t}^H$ will only increase the number of college graduates facing a binding credit constraint.

The return on college education has the following effect on the transfer b_t^C needed for graduating from college:

Proposition 4. *An increase in the return on college education $y_{2,t+1}^C/y_{2,t+1}^H$ decreases the transfer b_t^C needed in order to graduate from college.*

Proof. The partial derivative of b_t^C with respect to $y^C_{2,t+1}/y^H_{2,t+1}$ is equal to

$$\frac{\partial b_t^C}{\partial \frac{y^C_{2,t}}{y^H_{2,t}}} = \frac{\Gamma(y^H_{2,t+1} - g_2^H)(b_t^C - g_1^C)\left[(1-\tau)(Ry^H_{1,t} + y^H_{2,t+1}) - g_2^H + R(b_t^C - g_1^H)\right]}{(y^C_{2,t+1} - g_2^C)\left\{(1+\Gamma)R(b_t^C - g_1^C) - \left[(1-\tau)(Ry^H_{1,t} + y^H_{2,t+1}) - g_1^H + R(b_t^C - g_2^H\right]\right\}},$$

which following Lemma 1 is negative. □

Thus an increase in the return on college education decreases the transfer needed in order to graduate from college. It increases the value of having college education relative to the value of only high school education, making college education more attractive and thus the transfer needed in order to be indifferent between the two is lower. Hence there will be more agents receiving a transfer above this and more graduating from college. This is the larger the larger $y^H_{2,t+1}$ and b_t^C and the first period income of a high school graduate are. It is the smaller the larger $y^C_{2,t}$ and the difference $\bar{c}^C_{1,t} - c^{H*}_{1,t}$ are.

To summarise the results of the comparative static analysis: A decrease in the opportunity cost of education increases the number of graduates of a higher education degree. An increase in the return on education increases the number of graduates. A detailed analysis of the components can be found in Appendix A. If a decrease in the opportunity cost of education is due to a decrease in the wage of young dropouts, the increase in graduates will be entirely due to an increase of graduates facing a binding credit constraint, with the number of agents graduating that are not facing a binding credit constraint staying the same. This is due to the fact that a change in the wage of dropouts affects the thresholds $\{b_t^H, b_t^C\}$ above which agents graduate, but does not enter in the definitions (13) and (14) of the thresholds $\{\hat{b}_t^H, \hat{b}_t^C\}$ that govern whether or not the budget constraint of an agent is binding. If a decrease in the opportunity cost of education is due to an increase in the wage of young graduates, the total number graduates and the number of graduates not facing a binding credit constraint increases. The wage of young graduates also enters into (13) and (14), and thus $\{\hat{b}_t^H, \hat{b}_t^C\}$ are affected by a change of them. For the same reasons, an increase in the return on education that is due to a decrease in the wage of old drop outs increases the total number of graduates, but does not affect the number of graduates not facing a binding credit constraint. If an increase in the return on education are due to an increase in the wage of graduates, then the total number of graduates increases and the number of graduates facing a binding credit constraint increases, while the number of graduates not facing a binding credit constraint decreases.

Labour market polarisation means that the wages of middle skilled workers decrease and the wages of low and high skilled workers increase. As shown above, a decrease in the high school premium implies less mobility from the lower income level as b^H increases. It also implies an increase in the college premium, which implies that b^C decreases. Based on this comparative statics I expect to see a negative relationship between the opportunity cost of education and graduation rates, and a positive relationship between the returns on education and graduation rates. In the next Section, I will test these predictions on U. S. high school dropout rates.

5 Empirical Analysis

The model developed in this paper provides an interpretation of wage premiums as opportunity costs of education and returns on education. The comparative static analysis shows that an increase in the return on education increases educational attainment, whereas an increase in the opportunity costs of education decreases educational attainment. Any policy that is aimed at changing one wage differential must be financed and thus might have consequences for the other wage differential. Thus in order to make policy recommendations, I estimate their relative importance.

I focus on the drop out rate from high school, because as discussed in Section 1 high school education is important to explain differences in upward social mobility between countries, and as I expect mobility between regions for educational purposes to be less of an issue for high school education than for college education. I regress the drop out rate from high school on the wage premiums defined in the previous section in order to estimate the relative importance of these wage premiums. Analogous to Chetty et al. (2014a), I use commuting zones as defined by Tolbert and Sizer (1996) as unit of observations. I use the crosswalk files provided by Autor and Dorn (2013) to aggregate the average wages by education level, gender, and age as well as the high school dropout rate at commuting zone level.

For the wage data, I use the 1990 U. S. 5% census IPUMS. I define agents as "young" if they are of the age of 22 or below (graduating from high school at the age of 18 plus 4 year of college education). For the dropout of high school, the data come from the common core of data from the IES NCES in the year 2001.

I cannot directly observe the life period incomes $y_{1,t}$ or $y_{2,t+1}$ as defined in the model, nor the time spend in education μ, but I can observe the individual wage income in one year, which in the model is positively linearly related to the live period incomes. I use the average wage income by age group in one region in the census year w_1 and w_2 as a measure of life period income. As in the model, I define the opportunity cost of education as w_1^N/w_1^H, and the return on education as w_2^H/w_2^N. I regress these wage premiums by gender on the dropout rate $P(D)$ by gender, controlling for the logarithm of the average household income (I use the one derived by Chetty et al. (2014a)) in the commuting zone $\log(I_{HH})$ as a measure of overall wealth in the region. The result of this regression is presented in Table 5. I do not find any significant effect of wage premiums on the dropout rate for women, but for men I find a significant effect of both the opportunity cost of education and the return on education with the signs predicted by the model. A 10 percentage point increase in the wage differential between high school dropouts and high school graduates of young men (the opportunity cost of high school education) increases the high school dropout rate by 0.7 percentage points. A 10 percentage point increase in the wage differential between high school graduates and high school dropouts of old men (the return on education) decreases the high school dropout rate by 0.2 percentage points. This implies that changes in the relative wage of young graduates have a more than three times higher impact on the high school dropout rate.

6 Policy Analysis

I now use the model and the empirical results to draw some conclusions in order to propose policies that increase educational attainment of children from low income families. The analysis of the comparative statics of the model introduced in this paper suggests the following: One can increase the number of children

Table 5: *High School Dropout by Gender*

	(1) Men	(2) Women
w_1^N/w_1^H	0.0687***	-0.00154
	(0.0221)	(0.0170)
w_2^H/w_2^N	-0.0192***	-0.00362
	(0.00600)	(0.00296)
$\log(I_{HH})$	-0.0434***	-0.0231***
	(0.0105)	(0.00868)
Constant	0.527***	0.293***
	(0.114)	(0.0941)
Observations	207	207
R^2	0.155	0.034
Adjusted R^2	0.143	0.020

NOTE: Std. deviations in parenthesis
$^{*}p < 0.10$ $^{**}p < 0.05$ $^{***}p < 0.01$

attaining educational degrees by either decreasing the opportunity costs or by increasing the return on education. Both are possible by two ways, by either affecting the wages of those who drop out of education or by affecting the wages of those who attain education. A government could introduce policies that aim at decreasing the after tax income of dropouts $\{(1-\tau)y_{1,t}^N - g_1^N; (1-\tau)y_{2,t+1}^N - g_2^N\}$ by taxing these incomes higher (e. g. by introducing less progressive taxation schemes) and thus decreases the opportunity cost to education and increase the return on education. This would decrease the transfer needed in order to graduate from high school, but it would also decrease the income of poor parents and thus decrease the transfer received by poor children. It would also imply a redistribution of income from poor households to richer households.

Based on the empirical assessment, in which the opportunity cost to high school education had a much higher impact on high school dropout rates, I propose a different policy: Increasing the after tax income of young high school graduates $(1-\tau)y_{1,t}^H - g_1^H$ financed by decreasing the income of older high school graduates $(1-\tau)y_{2,t+1}^H - g_2^H$. This means that the opportunity cost of education decrease but also the return on education decreases.

This would not only increase the number of high school graduates but also increase the number of high school graduates that are not facing a binding credit constraint. As the government cannot directly affect the market wages w_1^H and w_2^H, such a policy could be achieved by increasing the subsidy for high school education h, resulting in a decrease in g_1^H, and increase the tax of high school graduates g_2^H.

Proposition 5. *An increase in the subsidy for high school education h^H (a decrease in g_1^H) payed by an increase in the taxes for old high school graduates g_2^H of equal size leads to a decrease in the transfer needed in order to graduate from high school and a decrease in the transfer needed in order to be not facing a binding credit constraint when graduating from high school $\hat{b}_t^H$.*

Proof. See Appendix B.

I find in the empirical analysis that the wage differential for young high school graduates has a more than three times higher impact on the high school dropout rate. At the same time, the wage level of young people is much lower than the one of old people, and also the period working being young is shorter, thus just a small percentage decrease in the return on education can cause a larger percentage decrease in the opportunity cost of education, leading to an overall decrease in the dropout rate. The advantage of this policy is also, that it does not affect life-time income, or redistribution between different income groups. The burden of subsidising high school education would fall on high school graduates.

I do not have data on the importance of wage differentials of college graduates, but the analysis of our model point to a similar policy of decreasing the return on education in favour of decreasing the opportunity cost of education in order to increase college graduation rates and decrease the number of college graduates not facing a binding credit constraint. This is especially relevant in the debate on tuition fees. Proponents of tuition fees for tertiary education argue that such tuition fees induce a payment by the beneficiary. This model suggests, that it might be preferable to pay for tertiary education after graduation, i. e. through higher taxes or publicly provided and subsidised student loans, and to subsidy tertiary education for poor students in order to decrease the opportunity costs of education. This might not only increase graduation rates of poor students, but also decrease the high levels of student debt in the U. S. and U. K.

Again, the government cannot directly change w_2^C, but it can set the education subsidy h^C and the tax rate for college graduates g_2^C. The suggested policy would be to finance an increase in h^C (a decrease in g_1^C) through an increase in g_2^C.

Proposition 6. *An increase in the subsidy for college education h^C (a decrease in g_1^C) payed by an increase in the taxes for old college graduates g_2^C leads to a decrease in the transfer needed in order to graduate from college and a decrease in the transfer needed in order to be not facing a binding credit constraint when graduating from college $\hat{b}_t^C$.*

Proof. See Appendix B.

Such a policy would increase the number of college graduates while decreasing the number of college graduates facing a binding credit constraint.

7 Conclusion

This paper investigates the effect of wage differentials on intergenerational income mobility by introducing a Galor and Zeira (1993) type OLG model with three levels of education where children make their own educational choices base on a transfer they receive from their parents. In this model, there are two forces that define the educational choices: the return on education and the opportunity costs to education.

In an empirical assessment of these two forces, I find that the opportunity cost of education have a much stronger influence on the probability to finish high school for men than the return on education. I use the conclusions drawn from the model to propose a policy which reduces the opportunity cost of education by subsidising education payed by taxes on older graduates. This policy has the advantage that it increases educational attainment of the poor, increases the number of graduates not facing a binding credit constraint, decreases income inequality due to age differences, while not implying changes in life-time income nor redistribution between education groups.

This paper argues that education and distributional policies should not only be concerned about their effect on the return on education, but also about their effect on the opportunity cost of education for the students. Policies that carefully manage age and educational premiums can improve educational outcomes for poor children, increasing upward mobility and intergenerational social mobility in general. It would be interesting to explore this insights further by modelling the role of ability into the model, which would allow to identify differences in the ability distribution of children with the same education level from different parental backgrounds. It would also be interesting to investigate gender differences in educational attainment. I do not find any significant effect of wage differentials on the high school dropout rate of women. It would be interesting to explore whether a similar model including endogenous fertility might provide an explanation of educational patterns of women.

References

Aaronson, Daniel and Bhashkar Mazumder, "Intergenerational Economic Mobility in the United States, 1940 to 2000," *The Journal of Human Resources*, 2008, *43* (1), 139–172.

Acemoglu, Daron and David Autor, "Skills, Tasks and Technologies: Implications for Employment and Earnings," in "Handbook of Labor Economics," Vol. 4b, Elsevier Inc., 2011, pp. 1043–1171.

Alonso-Carrera, Jaime, Jordi Caballé, and Xavier Raurich, "Fiscal policy, composition of intergenerational transfers, and income distribution," *Journal of Economic Behavior and Organization*, 2012, *84* (1), 62–84.

Alonso-carrera, Jaime, Xavier Raurich, and Jordi Caballé, "Intergenerational Mobility, Occupational Decisions and the Distribution of Wages," 2016.

Antonczyk, Dirk, Thomas DeLeire, and Bernd Fitzenberger, "Polarization and Rising Wage Inequality: Comparing the US and Germany," 2010.

Autor, David H and David Dorn, "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," *The American Economic Review*, 2013, *103* (5), 1553–1597.

Bailey, Martha J and Susan M Dynarski, "Inequality in Postsecondary Education," in Greg Duncan and Richard J. Murnane, eds., *Whither Opportunity? Rising Inequality, Schools and Children's Life Chances*, 2011, chapter 6, pp. 118–131.

Becker, Gary S. and Nigel Tomes, "Human Capital and the Rise and Fall of Families," *Journal of Labor Economics*, 1986, *4* (3), 1–39.

Checchi, Daniele, Andrea Ichino, and Aldo Rustichini, "More equal but less mobile? Education financing and intergenerational mobility in Italy and in the US," *Journal of Public Economics*, 1999, *74*, 351–393.

Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez, "Where is the land of Opportunity? The Geography of Intergenerational Mobility in the United States," *The Quarterly Journal of Economics*, 2014, *129* (4), 1553–1623.

_ , _ , _ , _ , and Nicholas Turner, "Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational Mobility," 2014.

Corak, Miles, "Do Poor Children Become Poor Adults? Lessons from a Cross-Country Comparison of Generational Earnings Mobility," *Research on Economic Inequality*, 2006, *13* (1993), 143–188.

_ , "Income Inequality, Equality of Opportunity, and Intergenerational Mobility," *Journal of Economic Perspectives*, 2013, *27* (3), 79–102.

_ , "Inequality from Generation to Generation: The United States in Comparison," in Robert S. Rycroft, ed., *The Economics of Inequality, Poverty, and Discrimination in the 21st Century*, 2013, chapter 6.

_ , Matthew J. Lindquist, and Bhashkar Mazumder, "A comparison of upward and downward intergenerational mobility in Canada, Sweden and the United States," *Labour Economics*, 2014, *30*, 185–200.

Couch, Kenneth A. and Dean R. Lillard, "Non-linear patterns of intergenerational mobility in Germany and the United States," in Miles Corak, ed., *Generational Income Mobility in North America and Europe*, 2004, chapter 8, pp. 190–206.

Deschênes, Olivier, "Unobserved Ability, Comparative Advantage, and the Rising Return to Education in the United States 1970-2000," 2001.

Eckstein, Zivi and Kenneth I. Wolpin, "Why Youths Drop Out of High School: The Impact of Preferences, Opportunities, and Abilities," *Econometrica*, 1999, *67* (6), 1295–1339.

Galor, Oded and Daniel Tsiddon, "Technological Progress and Economic Growth," *The American Economic Review*, 1997, *87* (3), 363–382.

_ and Joseph Zeira, "Income Distribution and Macroeconomics," *The Review of Economics Studies*, 1993, *60* (1), 35–52.

_ and Omer Moav, "Das Human Kapital: A Theory of the Demise of the Class Structure," *Review of Economic Studies*, 2006, *73*, 85–117.

Goos, Maarten, Alan Manning, and Anna Salomons, "Job polarization in Europe," *The American Economic Review: Papers and Proceedings*, 2009, *99* (2), 58–63.

Hassler, John and José V. Rodríguez Mora, "Intelligence, Social Mobility and Growth," *The American Economic Review*, 2000, *90* (4), 888–908.

_ , _ , and Joseph Zeira, "Inequality and mobility," *Journal of Economic Growth*, 2007, *12* (3), 235–259.

Hertz, Tom, "Trends in the Intergenerational Elasticity of Family Income in the United States," *Industrial Relations A Journal of Economy and Society*, 2007, *46* (1), 22–50.

Jäntti, Markus, Bernt Bratsberg, Knut Røed, Oddbjørn Raaum, Eva Österbacka, Anders Björklund, and Tor Eriksson, "American Exceptionalism in a New Light: A Comparison of Intergenerational Earnings

Mobility in the Nordic Countries, the United Kingdom and the United States," *IZA Discussion Paper*, 2006, (1938), 1–40.

Kornrich, Sabino and Frank Furstenberg, "Investing in Children: Changes in Parental Spending on Children, 1972-2007," *Demography*, 2013, *50* (1), 1–23.

Lee, Chul-In and Gary Solon, "Trends in Intergenerational Income Mobility," 2008.

Lemieux, Thomas, "Postsecondary Education and Increasing Wage Inequality," *The American Economic Review*, 2006, *96* (2), 195–199.

Lindley, Joanne and Stephen Machin, "Rising Wage Inequality and Postgraduate Education," 2011.

_ and _ , "The Quest for More and More Education: Implications for Social Mobility," *Fiscal Studies*, 2012, *33* (2), 265–286.

Manning, Alan, Marten Maarten Goos, and Alan Manning, "Lousy and Lovely Jobs: The Rising Polarization of Work in Britain ," *The Review of Economics and Statistics*, 2007, *89* (1), 118–133.

Mazumder, Bhashkar, "Fortunate Sons: New Estimates of Intergenerational Mobility in the United States Using Social Security Earnings Data," *The Review of Economics and Statistics*, 2005, *87* (2), 235–255.

McNeal, R B, "Are students being pulled out of High School? The effect of adolescent employment on dropping out," *Sociology of Education*, 1997, *70* (3328), 206–220.

Moav, Omer and Oded Galor, "From Physical to Human Capital Accumulation," *The Review of Economic Studies*, 2004, *71* (4), 1001–1026.

Pereira, Pedro Telhado and Santiago Budría, "Educational Qualifications and Wage Inequality: Evidence for Europe," 2005.

Piketty, Thomas, "Social Mobility and Redistributive Politics," *The Quarterly Journal of Economics*, 1995, *110* (3), 551–584.

_ , *Le capital au XXIe siècle*, Seuil, 2013.

Putnam, Robert D, Carl B Frederick, and Kaisa Snellman, "Growing Class Gaps in Social Connectedness among American Youth," 2012.

Ramey, Garey and Valerie A. Ramey, "The Rug Rat Race," *Brookings Papers on Economic Activity*, 2010, *2010* (1), 129–176.

Restuccia, Diego and Carlos Urrutia, "Persistence of Earnings: Intergenerational The Role of Early and College Education," *The American Economic Review*, 2004, *94* (5), 1354–1378.

Solon, Gary, "Intergenerational mobility in the labor market," in "Handbook of Labor Economics," Vol. 3 1999, pp. 1761–1800.

Tolbert, Charles M. and Molly Sizer, "U.S. Commuting Zones and Labor Market Areas: A 1990 update," 1996.

A Detailed Comparative Analysis

A decrease in the opportunity cost of high school education can be caused by an increase in $y_{1,t}^H$ or by a decrease in $y_{1,t}^N$. I first assume that the change in the wage premium is caused by a decrease in $y_{1,t}^N$.

Proposition 7. *A decrease in the income of young high school dropouts $y_{1,t}^N$ decreases b_t^H.*

Proof. The partial derivative of b_t^H with respect to $y_{1,t}^N$ is equal to

$$\frac{\partial b_t^H}{\partial y_{1,t}^N} = -\frac{(1-\tau)R\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right]}{(1+\Gamma)R\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right] - \left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]},$$

which following Lemma 1 is positive. □

The denominator is equal to the difference $\bar{c}_{1,t}^H - c_{1,t}^{N*}$. Thus the higher the difference $\bar{c}_{1,t}^H - c_{1,t}^{N*}$ and the lower the constraint first period consumption of a high school graduate, the higher will be the decrease in b_t^h. The cut-off value for facing a binding credit constraint $\hat{b}_t^H$ is not affected by a change in $y_{1,t}^N$. Thus if it decreases, more students facing a binding credit constraint will graduate from high school.

I now assume that the change in the opportunity cost of high school education is caused by an increase in $y_{1,t}^H$ and that $y_{1,t}^N$ stays constant.

Proposition 8. *An increase in the income of young high school graduates $y_{1,t}^H$ decreases b_t^H and $\hat{b}_t^H$.*

Proof. The partial derivative of b_t^H with respect to $y_{1,t}^H$ is equal to

$$\frac{\partial b_t^H}{\partial y_{1,t}^H} = \frac{(1-\tau)\left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]}{(1+\Gamma)R\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right] - \left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]} \qquad (17)$$

which following Lemma 1 is negative. One can derive the partial derivative of $\hat{b}_t^H$ with respect to $y_{1,t}^H$

$$\begin{aligned}\frac{\partial \hat{b}_t^H}{\partial y_{1,t}^H} &= \frac{1-\tau}{\Gamma} - \frac{(1-\tau)(1+\Gamma)}{\Gamma} \\ &= -(1-\tau),\end{aligned}$$

which is also negative. □

The denominator of (17) is equal to the the difference $\bar{c}_{1,t}^H - c_{1,t}^{N*}$ times $\bar{c}_{1,t}^H$. Thus the larger the difference between constraint and unconstrained consumption and the larger the constraint first period consumption, the smaller the effect of $y_{1,t}^H$ on b_t^H. The nominator is equal to $\bar{c}_{1,t}^H c_{1,t}^{N*}$. The higher $\bar{c}_{1,t}^H$ and $c_{1,t}^{N*}$, the larger is the increase. Thus a increase in $y_{1,t}^H$ increases the value of $\hat{b}_t^H$. It will increase the number of student graduating from high school, and increase the number of students that are not facing a binding credit constraint when graduating from high school.

Thus from a policy perspective, a decrease in $y_{1,t}^N$ might increase the number of high school graduates, but only the number of graduates facing a binding credit constraint, whereas an increase in $y_{1,t}^H$ increases the number of high school graduates and increases the number of high school graduates that are not bound by the credit constraint.

As in the case of the opportunity cost of education, the increase in the return on high school education can come from a decrease in $y^N_{2,t+1}$ or from an increase in $y^H_{2,t+1}$.

Proposition 9. *A decrease in the wages of old high school dropouts $y^N_{2,t+1}$ decreases the transfer b^H_t needed in order to graduate from high school.*

Proof. The partial derivative of b^H_t with respect to $y^N_{2,t+1}$ is equal to

$$\frac{\partial b^H_t}{\partial y^N_{2,t+1}} = -\frac{(1-\tau)\left[(1-\tau)y^H_{1,t} - g^H_1 + b^H_t\right]}{(1+\Gamma)R\left[(1-\tau)y^H_{1,t} - g^H_1 + b^H_t\right] - \left[(1-\tau)(Ry^N_{1,t} + y^N_{2,t}) - g^N_2 + R(b^H_t - g^N_1)\right]}$$

which following Lemma 1 is positive. □

This expression is increasing in the first period consumption of high school graduates and decreasing in the difference between $\bar{c}^H_{1,t}$ and $c^{N*}_{1,t}$. $y^N_{2,t+1}$ has no effect on $\hat{b}^H_t$, thus a decrease in $y^N_{2,t+1}$ will increase the number of high school graduates that graduate while facing a binding credit constraint.

Proposition 10. *An increase in the income of old high school graduates $y^H_{2,t+1}$ decreases the transfer b^H_t needed in order to graduate from high school but increases the transfer needed in order to not face a binding credit constraint when acquiring high school education $\hat{b}^H_t$.*

Proof. The partial derivative of b^H_t with respect to $y^H_{2,t+1}$ is equal to

$$\frac{\partial b^H_t}{\partial y^H_{2,t+1}} = \frac{\Gamma\left[(1-\tau)y^H_{1,t} - g^H_1 + b^H_t\right]\left[(1-\tau)(Ry^N_{1,t} + y^N_{2,t}) - g^N_2 + R(b^H_t - g^N_1)\right]}{(y^H_{2,t+1} - g^H_2)\left\{(1+\Gamma)R\left[(1-\tau)y^H_{1,t} - g^H_1 + b^H_t\right] - \left[(1-\tau)(Ry^N_{1,t} + y^N_{2,t}) - g^N_2 + R(b^H_t - g^N_1)\right]\right\}}$$

which following Lemma 1 is negative. The partial derivative of $\hat{b}^H_t$ with respect to $y^H_{2,t}$ is equal to

$$\frac{\partial \hat{b}^H_t}{\partial y^H_{2,t+1}} = \frac{1-\tau}{R\Gamma}$$

which is positive. □

This negative effect on b^H_t is the larger, the larger first period income of dropouts and high school graduates, the lower the difference $\bar{c}^H_{1,t} - c^{N*}_{1,t}$ and the lower $y^H_{2,t+1}$ itself is. Thus an increase in $y^H_{2,t+1}$ increases the number of high school graduates, but also increases the number of high school graduates that are facing a binding credit constraint.

The return on college education can increase because the income of old high school graduates decreases, or because the income of college graduates increases. A change in the income of high school graduates has the following direct effect on b^C_t:

Proposition 11. *A decrease in the income of high school graduates $y^H_{2,t+1}$ decreases the transfer b^C_t needed in order to graduate from college.*

Proof. The partial derivative of b_t^C with respect to $y_{2,t+1}^H$ is equal to

$$\frac{\partial b_t^C}{\partial y_{2,t+1}^H} = -\frac{(1-\tau)(b_t^C - g_1^C)}{(1+\Gamma)R(b_t^C - g_1^C) - \left[(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - g_1^H + R(b_t^C - g_2^H\right]}$$

which following Lemma 1 is positive. □

A decrease in $y_{2,t+1}^H$ will decrease b_t^C and thus increases the number of college graduates. This increase will be the larger, the larger b_t^C and the smaller $\bar{c}_{1,t}^C - c_{1,t}^{H*}$. As it has no effects on $\hat{b}_t^C$, it will only increase the number of credit constrained college graduates, but not the number of credit-non constrained college graduates.

Proposition 12. *An increase in the income of college graduates $y_{2,t+1}^C$ decreases the transfer b_t^C needed in order to graduate from college and increases the transfer $\hat{b}_t^C$ needed in order to be not facing a binding credit constraint when graduating from college.*

Proof. The partial derivative of b_t^C with respect to $y_{2,t+1}^C$ is equal to

$$\frac{\partial b_t^C}{\partial y_{2,t+1}^C} = \frac{\Gamma(b_t^C - g_1^C)\left[(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - g_1^H + R(b_t^C - g_2^H\right]}{(y_{2,t+1}^C - g_2^C)\left\{(1+\Gamma)R(b_t^C - g_1^C) - \left[(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - g_1^H + R(b_t^C - g_2^H\right]\right\}}$$

which following Lemma 1 is negative. The partial derivative of $\hat{b}_t^C$ with respect to $y_{2,t+1}^C$ is equal to

$$\frac{\partial \hat{b}_t^C}{\partial y_{2,t}^C} = \frac{1-\tau}{R\Gamma},$$

which is positive. □

An increase in $y_{2,t}^C$ decreases b_t^C and thus increases the number of college graduates. The effect is the larger, the larger life time income of high school graduates, the larger b_t^C, and the smaller $\bar{c}_{1,t}^C - c_{1,t}^{H*}$ and $y_{2,t+1}^C$. A change in $y_{2,t+1}^C$ does not only affect b_t^C but also the cut-off value below which college graduates are facing a binding credit constraint. Thus an increase in $y_{2,t+1}^C$ increases the number of college graduates in total, and the number of college graduates facing a binding credit constraint, but decreases the number of college graduates not facing a binding credit constraint.

B Proofs of Policy Analysis

Proof of Proposition 5. The partial derivative of b_t^H with respect to g_1^H is

$$\frac{\partial b_t^H}{\partial g_1^H} = -\frac{(1+\Gamma)R\left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]}{(1+\Gamma)R\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right] - \left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]}$$

which following Lemma 1 is positive. The partial derivative of b_t^H with respect to g_2^H is equal to

$$\frac{\partial b_t^H}{\partial g_2^H} = -\frac{\Gamma\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right]\left[(t-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]}{\left[(1-\tau)y_{2,t+1}^H - g_2^H\right]\left\{(1+\Gamma)R\left[(1-\tau)y_{1,t}^H - g_1^H + b_t^H\right] - \left[(1-\tau)(Ry_{1,t}^N + y_{2,t+1}^N) - g_2^N + R(b_t^H - g_1^N)\right]\right\}}$$

which following Lemma 1 is also positive. As

$$\frac{\partial b_t^H}{\partial g_2^H} = \frac{\Gamma}{R(1+\Gamma)}\frac{(1-\tau)y_{1,t}^H - g_1^H + b_t^H}{(1-\tau)y_{2,t+1}^H - g_2^H}\frac{\partial b_t^H}{\partial g_1^H}$$

and according to equation (6) it is true that

$$\frac{\Gamma}{R(1+\Gamma)}\frac{(1-\tau)y_{1,t}^H - g_1^H + b_t^H}{(1-\tau)y_{2,t+1}^H - g_2^H} < 1,$$

$\partial b_t^H / \partial g_1^H > \partial b_t^H / \partial g_2^H$ and a simultaneous increase in g_2^H and decrease in g_1^H of equal size decreases b_t^H. As

$$\frac{\partial \hat{b}_t^H}{\partial g_1^H} = \frac{1}{\Gamma} - \frac{1+\Gamma}{\Gamma} = -1$$

and

$$\frac{\partial \hat{b}_t^H}{\partial g_2^H} = \frac{1}{R\Gamma}$$

this policy also decreases $\hat{b}_t^H$. □

Proof of Proposition 6. The partial derivative of b_t^C with respect to g_1^C is

$$\frac{\partial b_t^C}{\partial g_1^C} = -\frac{(1+\Gamma)R\left[(Ry_{1,t}^H + y_{2,t+1}^H) - g_2^H + R(b_t^C - g_1^H)\right]}{(1+\Gamma)R(b_t^C - g_1^C) - \left[(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - g_2^H + R(b_t^C - g_1^H)\right]}$$

which following Lemma 1 is positive. The partial derivative of b_t^C with respect to g_1^C is equal to

$$\frac{\partial b_t^C}{\partial g_1^C} = -\frac{\Gamma(b_t^C - g_1^C)\left[(Ry_{1,t}^H + y_{2,t+1}^H) - g_2^H + R(b_t^C - g_1^H)\right]}{\left[(1-\tau)y_{2,t+1}^C - g_2^C\right]\left\{(1+\Gamma)R(b_t^C - g_1^C) - \left[(1-\tau)(Ry_{1,t}^H + y_{2,t+1}^H) - g_2^H + R(b_t^C - g_1^H)\right]\right\}}$$

which following Lemma 1 is also positive. It follows from this that

$$\frac{\partial b_t^C}{\partial g_2^C} = \frac{\Gamma}{R(1+\Gamma)}\frac{b_t^C - g_1^C}{(1-\tau)y_{2,t+1}^H - g_2^C}\frac{\partial b_t^C}{\partial g_1^C}.$$

According to equation (7) it is true that

$$\frac{\Gamma}{R(1+\Gamma)}\frac{b_t^C - g_1^C}{(1-\tau)y_{2,t+1}^H - g_2^C} < 1.$$

Thus $\partial b_t^C / \partial g_1^C > \partial b_t^C / \partial g_2^C$ and a simultaneous increase in g_2^C and decrease in g_1^C of the same size decreases b_t^C. As

$$\frac{\partial \hat{b}_t^C}{\partial g_1^C} = \frac{1}{\Gamma} - \frac{1+\Gamma}{\Gamma} = -1$$

and

$$\frac{\partial \hat{b}_t^C}{\partial g_2^C} = \frac{1}{R\Gamma},$$

this policy also decreases $\hat{b}_t^C$. □